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THE MATHEMATICS TEACHER

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WHY STUDENTS FAIL IN MATHEMATICS.

BY HELEN A. MERRILL.

This subject was suggested some weeks ago, at a meeting of the Council of this Association, as one in which both school and college teachers are likely to be interested. There is a certain advantage in directing the attention occasionally to facts with which all are perfectly familiar, particularly if those same facts involve problems which are never completely solved, but which must be faced anew with each new set of students that enters our classrooms. This is my reason for undertaking today to open a discussion of a subject with which all of us may well be equally familiar. I am further encouraged by a remark made last fall by Professor (now Major) Julian Coolidge, who told an audience that he was addressing on a rather elementary subject, that while he had often been bored by too difficult mathematical discussions, he had never yet been bored by too easy ones, a sentiment in which every hearer probably concurred.

This subject enjoys the marked advantage of needing no definitions of terms, no proof of the claim which it implies. We all know what failures in mathematics, technically so called, mean, and that students make these failures.

Statistics as to the number of failures in mathematics and other subjects have been published in many school and college reports, in educational journals, etc. Into this side of the question I do not propose to enter, though it is one that interests me greatly, and I have for many years collected averages and percentages for private use only. I assume without attempt at proof that the average of failures is rather high, probably higher than in any other subject, and suggest possible reasons, which may well be added to by others, with the hope that remedies also may be suggested. For there is little reason for scrutinizing the dark side of any subject, unless some means be thereby found of brightening that side, not by whitewashing it, but by removing the dust and stains of neglect and ignorance.

I propose to speak under four general heads of the main reasons for failures in the study of mathematics, asking successively how much of the failure may be due to the subject itself, to those who teach it, to parents and friends of students, and finally to the students themselves.

That mathematics is a difficult subject no one can deny. It demands, among other things, accuracy of thought and statement, definite mental concepts, connected thinking, a fair memory, quickness to recognize relations between forms and numbers, power of generalization, a willingness to work hard. The abstractness of much of the work renders it difficult to most minds, and obnoxious to some. The ideal elements that are the object of study seem remote and unreal. This troubles those who like at each step to feel the solid ground under their feet, while those who revel in flights of fancy are continually being pulled to earth by the insistent *why* and the demand for obedience to the most important laws of logic. There is a new language to be learned, new symbols to be manipulated, new elements to be dealt with, in some cases a queer upsetting and rearranging of one's mental processes. The ability to reason about certain subjects seems, oddly enough, not always to ensure that ability in mathematics, the thought processes are different, highly specialized, and subject to rigid rules, the topics of thought seem remote from everyday thinking and experience, impersonal and detached, and at certain ages students demand intense human interests, the object of the work as a whole is not readily seen. We sometimes hear the ability to put 2 and 2 together referred to as a rather rare gift, and the thought processes required to draw even a very simple conclusion from given algebraic or geometric premises is more complicated than we sometimes think. In a book by Benchara Branford entitled

"A Study of Mathematical Education," there is an interesting and instructive account of the author's methods of work in geometry with quite young children. One intelligent child could readily find by superposition that the triangle *A* was exactly equal to the triangle *B* and to the triangle *C*, but did not suspect that this discovery involved any relation between *B* and *C*. In fact the teacher was unable to make this relation clear by any method but the superposition of *B* and *C*.

The very definitions of mathematics that have been suggested often emphasize the difficulty of approach for the average youthful mind. The abstract quality of the subjects studied is evident in all, while in some of them is stated or implied that this science involves the drawing of necessary conclusions. But these conclusions are in many instances *necessary* only to the mind that is sufficiently trained in drawing them to feel their necessity. To the bewildered student, groping painfully from point to point, the triumphant *therefore* does not always mean that the goal is necessarily attained; in fact, we may at times strongly suspect that a *because* or an *and* would be accepted with equal resignation and seem quite as plausible.

These are some of the many characteristics that make mathematics hard. Of course it is hardly necessary to say that many of these very characteristics make it attractive and fascinating to some types of mind. The fact that it is abstract and impersonal, that it requires effort and compels concentration has made it a welcome refuge to minds distracted by the perplexities of everyday life. Probably every one of us knows the joy of losing ourselves and our cares completely in a problem that taxes our minds. How shall we learn to appreciate the confusion of minds lost in a sea of vague, elusive ideas, never quite grasped, finding in even the simplest piece of work pitfalls whose existence we never imagined, reading into what seem to us plain and obvious statements most extraordinary meanings, the evolving of which would seem to be a tax to any brain! We must all have been astounded at the ability of the dull or average student to misinterpret the simplest statements, an ability that makes it necessary to look at every question on an examination paper from all possible, or even impossible, angles.

But these considerations are leading us away from mathematics proper to those who are teaching it, and our second topic is already before us.

How far can the blame for failures in mathematics be laid upon the teachers? None of us will claim that teachers are faultless, nor are we ready to admit meekly that all the fault lies here.

The elementary mathematics have been taught for so long that the method, as well as their content, has been quite fully standardized, and while no satisfactory measurements can be made, it is probable that mathematics is taught in schools and colleges at least as well as any other subject. The last issue of THE MATHEMATICS TEACHER calls attention to what it terms "a deplorable condition in five states," the statistics showing that one out of every five high school teachers has had nothing above normal school training, while some have not even high school training. New England figures of course show a very different state of affairs, but many students in New England come from other states and thus their problem becomes our problem. However little poor teaching of mathematics there may be, there is certainly more than there should be. Even when high school teachers are listed as college graduates, it by no means follows that they have had more than one year of college mathematics. Whatever efforts mathematics dpartments may make, refusing to recommend unfit candidates, and trying to keep students from entering fields for which they have little preparation, teachers' agencies and even school superintendents often seem possessed of the idea that a B.A. implies the ability to teach any high school subject, and the program of a teacher of Latin or English is completed by adding a mathematics class, generally in beginning algebra, for there seems to be a rather widely prevalent notion that anyone can teach that. It is claimed that this work cannot be avoided, it is a choice between teaching this class and declining the position, and urgent advice to refuse to teach any subject in which one is not at least three years ahead of one's class is apt to be regarded by principals and teachers alike as impractical.

I do not mean to claim that all such teaching is a failure, but I do claim, and that emphatically, that this unhappy fact accounts in part for the attitude of some students toward mathematics. I have been told by some few students that never until they entered college had they been taught by any one who showed any enthusiasm for the subject, and the fact that mathematics has any qualities capable of inspiring enthusiasm was to them a quite new idea. Such students have been cheated of one of their rights—the proper approach to a great subject. To the teaching of mathematics quite as much as to any subject is that homely old saying applicable, that there must be more than a quart of molasses in a jug if you are to get a quart out. Or, to use a different figure, the right guide for young travellers through a wide-stretching land is surely not the one who knows just one path leading only a few miles into the country. Even if he knows that short stretch of the way rather well, how is he to point out to his often slow and reluctant followers the interesting side paths, and occasional far-reaching glimpses.

A great amount of mathematical knowledge does not of course ensure first class teaching, many other elements enter in, but, other things being equal, all the chances of success are on the side of the one who can look at each day's work from many different points of view, who can estimate the value of each part of the subject, not according to its bearing upon the coming examination, but according to its value for mental development and its later usefulness.

When the student's preparation for college is plainly at fault, the lack is apt to be in one of a few directions. Probably there has been insufficient drill on the fundamentals, due either to a failure to appreciate their importance or to lack of time. The school year has grown shorter, and many more subjects are clamoring for the student's attention. Small wonder that teachers everywhere are conscious of the necessity of cutting down to the smallest possible limit the amount of time to be assigned to each topic. It takes a far-sighted and experienced teacher to choose wisely between the topics that may safely be given less consideration, and those that must be drilled into the students' minds until they become second nature, whatever the cost in time may be. Sometimes too elementary a text has been used, and the mental muscles of the student have not been developed. But with the right kind of teacher the text-book is relatively unimportant, and the most up-to-date, all-inclusive, pedagogically perfect text cannot be guaranteed as fool-proof.

Sometimes the numbers in the class have been so large that the student has not received the individual attention needed. This lack displays itself chiefly in untidy or badly arranged work, which as a rule indicates the need of mental discipline.

Sometimes the student has suffered from being personally conducted over every difficulty, with every possible thorn and stone removed from the path, and assistance rendered at every turn. Such a student is likely to be handicapped in later work, it is hard to throw away the crutches upon which one has been encouraged to lean. One hears with dismay the demand for private tutoring when difficulties appear, and wonders whether the schools are fostering such aids as much as seems to be indicated. When a good part of the fun of mathematics lies in doing one's own thinking, it is unfortunate that any student should fail to find, or at least to be encouraged to find, this pleasure in independent work.

It is quite possible for prospective teachers to spend so long a time in study that they get out of touch with young students The difficulties which they encounter are far removed from our experience, the subjects look so absurdly easy. We are led again to the question which teachers can hardly put to themselves too often:—How shall we learn to see clearly our pupils' difficulties? How can we put ourselves in their place and so learn to forestall their blunderings, and make the subject—not necessarily easy, but clear?

Any one who is teaching a child to read would do well at times, when progress seems to be slow, to try reading aloud from a book held upside down. The letters and words will probably have a far more familiar look to him than the ordinary page will have to the child. Some such experience is often illuminating; in fact, one of the chief safeguards against impatience or discouragement or lack of sympathy with our students is to give ourselves a chance occasionally to show how stupid we can be. We need to be working on something new and hard all the while that we are engaged in teaching, and, unless our minds are quite exceptional, we shall be taught thereby how easy it is to jump at wrong conclusions, to miss the point of the argument, to fail to see something which we finally discover to be surprisingly obvious.

There is nothing easier than to give instructions, all-inclusive and infallible, for success in teaching. You have probably read these two rules, guaranteed to make any one who follows them successful:

Rule 1.—Know everything there is to be known about the subject.

Rule 2.—Arouse in the mind of each student an eager desire and determination to know all there is to be known about the subject.

Like all really good nonsense, this has a large modicum of common sense in it. Our failures as teachers can probably be classed more or less roughly under two heads—failure to know enough about the subject, its history, its connection with other subjects, its interesting applications, the relative importance of its various subdivisions and the best methods of presenting them; and failure to give ourselves wholeheartedly to our teaching, to enjoy it so thoroughly, to appreciate it so keenly that our students cannot fail to be influenced by our enthusiasm.

No student ought to complete a course in mathematics without the feeling that there must be something in it, without catching a glimpse, however fleeting, of its possibilities, without at least a few moments of pleasure in achievement and insight.

The third reason for failure may be disposed of more briefly. How much have one's family or friends or the outside non-mathematical public to do with this matter? It is often a wonder that the modern boy and girl find time to do any studying. Many parents resent any attempt to have work brought home. Music lessons, dancing lessons, etc., take much time, and young folks are allowed an extraordinary amount of social diversion, with late hours and excitement telling sadly on the next day's work. This however must affect one study about as much as another, and while it doubtless helps to account for some failures, it has no exclusive relation to mathematics. I wish to speak of a certain psychological influence which has a marked effect upon our mathematical students especially.

It is clearly impossible to arrange a scale of hardness in studies such as is used in mineralogical tests. But if the formation of such a scale were attempted, mathematics would probably head most of the lists. Once label a subject *Very hard,* and let that label be flaunted before the young pupils' sight, and they are handicapped from the start. They magnify every difficulty, are discouraged too easily, accept failures as all but inevitable. This disadvantage works in many ways. Children are pitied for having to work hard examples, they are made to tremble at the very thought of algebra and geometry. If they express any pleasure in the subject they are called grinds, or sharks, or are told "Just wait till you get to radicals." Students who have just finished a course in algebra and geometry delight in terrifying those in the class below them, exaggerating its difficulties, discouraging them from reasonable efforts to succeed by instilling a belief in the futility of such attempts, magnifying the slaughter wrought by examinations, or perhaps declaring that the only way they themselves got through was by committing all the proofs to memory, a tale which can rarely be true, but which is often swallowed with avidity. If it were possible to eliminate from the young minds, that cling so tenaciously to some forms of tradition, this conventional view of mathematics, I believe that we should find pleasure in learning and in teaching mathematics wonderfully increased, and failures in the subject correspondingly diminished. Is there any way in which we can achieve this? It is worth much thought and effort.

The general attitude toward mathematics for girls is peculiarly hampering to them. I often marvel, not that some fail, but that so large a proportion succeed, when fathers and mothers say to me: "It seems such a pity for girls to have to waste their time on mathematics. It is always hard for them, and why should they be obliged to study it?" And then they often add "It isn't as if my daughter would ever have to earn her own living," their theory evidently bring that the only possible reason why a girl should study mathematics is in order to teach it to others, who will have no mortal use for it except to teach it to other poor victims, etc.

A not at all bloodthirsty school principal once remarked: "I could have an excellent school if I could kill off all the parents." Many of us may have had a similar feeling. But it is not the parents only. Why should we blame them, when high educational officials go about the country proclaiming: "Why teach girls algebra?", "Why compel students to take any mathematics beyond arithmetic?"

Here again the fault is in part in the teaching. Have we allowed the work to become too formal, too theoretical, too removed from everyday use? If we could in some way help the students, their parents, the general public, to see what mathematics really is like, and what it can do, would not some of this outcry cease?

Some efforts to bring mathematics up to date are frankly amusing. Whether the substitution of Zeppelins and airships for the old-time fox and greyhound or couriers makes the subject seem more vital, some who are here may be able to say. Letting the pipes which in Hero's days emptied into a cistern now convey gasoline into an automobile tank may make them easier to reckon with. Perhaps the change of name helps the pain.

But if we can in some way help all who have an interest in our students to see that mathematics is of service to them, not alone in terms of dollars and cents, though we readily grant that type of usefulness, but as an aid in gaining powers and qualities that all wish our young folks to acquire, this cause of failure in mathematics will be removed.

And finally, what of the students themselves? How far are they to be blamed for lack of success?

Individual failures are usually to be ascribed to one or more of three lacks—lack of ability, lack of training, lack of effort. The first may be due to the grandparents, the second may be the fault of teachers or parents, the third in the ultimate analysis must come home to the student, and it is the one that counts for the most. A dull or slow student may succeed through hard work, poor preparation may be offset by hard work, but persistent unwillingness to work cannot long be counterbalanced by natural ability or thorough preparation.

Are there any boys and girls who really cannot do mathematics? I presume that there are. In a western high school, which has a fine reputation for its mathematical work, I visited a class composed of boys and girls who, after many attempts on the part of their teachers, and possibly on their own part, seemed quite unable to do the ordinary work. So an attempt was being made to give them very simple and very practical work in arithmetic, for which they might find ready use. It had, however, proved nearly if not quite as hard for them to get on in this class. It was a small class, but the amount of stupidity registered in their faces would have been a generous allowance for a very large class. They had the look of being quite unequal to any intellectual effort, but they were strong, husky looking boys and girls, and should have been put soon to some form of manual labor, which might perhaps develop their minds along with their bodies.

But boys and girls of mediocre ability can surely do the amount of mathematics ordinarily required of them in school if they have pluck and determination, while it seems likely that students who have sufficient intelligence to warrant their being in college, who are what is known as "college material," can do as much mathematics as is ever required in those colleges which still hold to the prescribed course. Failure in these elementary courses is rarely due to actual inability to do the work. The difficulty is that into the bright lexicon of youth has crept a word that was once supposed not to be found there.

A colleague in our Latin department said to me lately: "I can tell you in one word why students fail in mathematics and in Latin prose composition—*obstinacy*." I repeated this later to a group of students, rather expecting indignant denial, but to my surprise they agreed heartily.

Anticipation of difficulty and the psychological influence already referred to have made them accept failure without a struggle, and, having thus resigned themselves, it is most difficult to arouse them to genuine and determined effort. Their attitude is well expressed in the words of the foreigner who had fallen into the water, "I will drown, nobody shall help me."

It has been said very truly that "Can't do mathematics" generally means "won't use paper and pencil." There is a mental inertia which makes it possible for students to sit and stare at a set of equations, and wonder how the result ever comes out, without once trying to see what would happen if they themselves carried out the processes suggested. Of course these are the very ones who will complain most bitterly of the amount of time they have spent on the assignment.

There are also the students who are willing to spend time, but are strangely reluctant to spend brain energy. With what a sense of virtue the young student displays three pages of figures devoted to working an example for which a few lines are ample! Students who begin to see the charm of short cuts have often had their first introduction to the pleasure of mathematical thought.

It is easy to pile up charges against our unsatisfactory students; they are easily discouraged, they do not know how to apply themselves, they have no power of concentration, they have never learned the joy of hard work, they are too ready to ascribe their failures to causes outside themselves. (Note in this connection that in their vocabulary *flunk* is apt to be a transitive verb) they wish a painless education, they are bored by insistent demands for accuracy and regularity of work.

All this seems to indicate weakness of *spirit* rather than of mind, and it may seem to some of us that there is no cure comparable to a stiff course of mathematics. Certainly we may rejoice in the conviction that, even at the cost of some failures, many students have learned by this discipline lessons that have later made possible successes in many directions.

A recent report on business failures for the last three years shows that the number of failures each year is about 75 per cent. of the number of new firms starting in business. It may be a comfort at this point to consider how much more successful these young students are than business men For, of the new students that come yearly to our schools and colleges, surely the percentage of failures never reaches 75. In point of fact the great majority do good work, while a very large majority pass in all their mathematical subjects.

And so we may be sure that, despite the difficulties inherent in mathematics, they are not too great for our students to cope with them successfully; despite the fact that all of us who are teaching mathematics are not doing it perfectly, we really are meeting with some success; despite the evil reports of some unwise and ignorant critics, their influence is not all-powerful; and despite the faults and follies of some few students, the large majority work with spirit and resolution.

For the sake of our students, and no less for the sake of our chosen subject, we must work to lessen the number of preventable failures, never by lowering our standards, but rather by raising our ideals for teachers and students alike, so that all who teach mathematics shall be equipped with wisdom and patience and insight and enthusiasm, and shall be able not only to hold their students to regular, accurate, thorough work, but also to inspire them all along their course with the interest, the charm, and the far-reaching possibilities of this study.

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